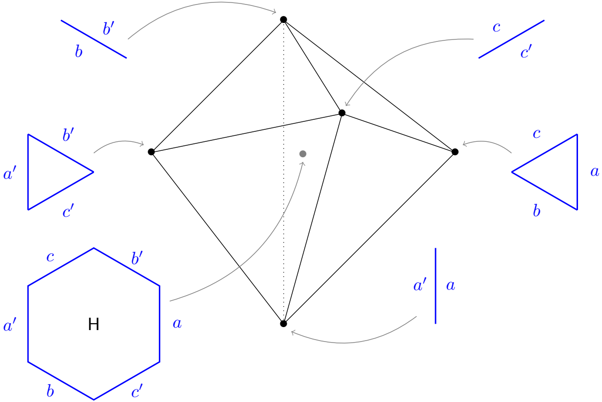
Minkowski indecomposability of polytopes

Germain Poullot with Arnau Padrol

ArXiv: 2512.05307



10 December 2025

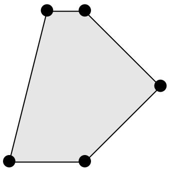
1. What is a polytope?
    - Two definitions of polytopes
    - Faces and normal fans
2. Deformations (weak Minkowski summands)
    - Minkowski decomposition
    - Cone of deformations
3. Graph of edge dependencies
    - Triangle, parallelograms
    - Graph of edge dependencies
    - Applications

What is a polytope?

Polytopes

Definition

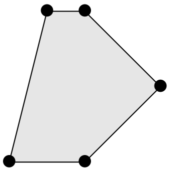
Polytope: convex hull of finitely many points in $\mathbb{R}^n$



Polytopes

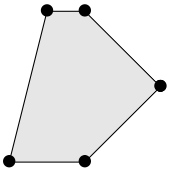
Definition

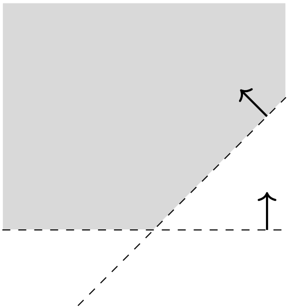
Polytope: convex hull of finitely many points in $\mathbb{R}^n$
bounded intersection of finitly many half-spaces in $\mathbb{R}^n$



Definition

Polytope: convex hull of finitely many points in $\mathbb{R}^n$
bounded intersection of finitly many half-spaces in $\mathbb{R}^n$

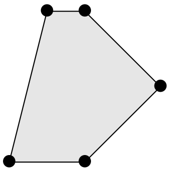


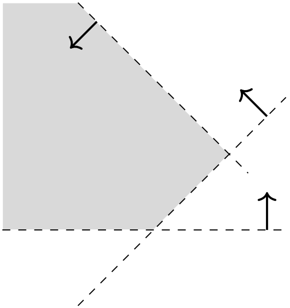


Polytopes

Definition

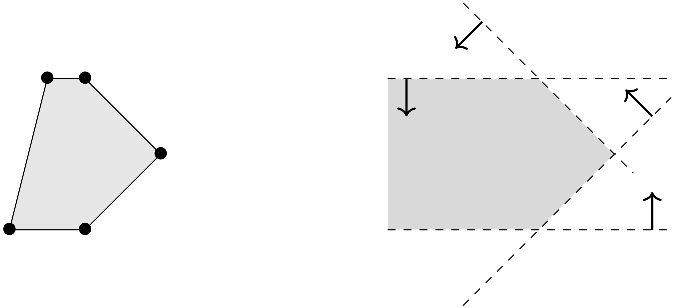
Polytope: convex hull of finitely many points in $\mathbb{R}^n$
bounded intersection of finitly many half-spaces in $\mathbb{R}^n$





Definition

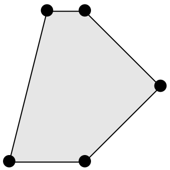
Polytope: convex hull of finitely many points in $\mathbb{R}^n$
bounded intersection of finitly many half-spaces in $\mathbb{R}^n$

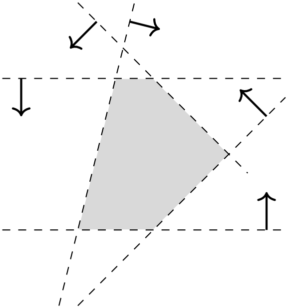


Polytopes

Definition

Polytope: convex hull of finitely many points in $\mathbb{R}^n$
bounded intersection of finitly many half-spaces in $\mathbb{R}^n$

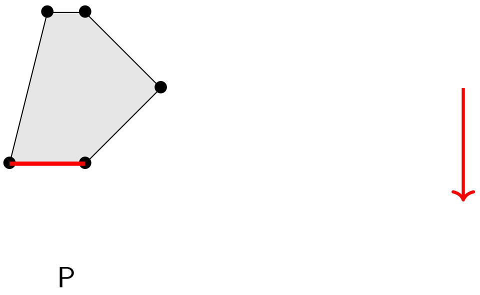




Faces

Definition

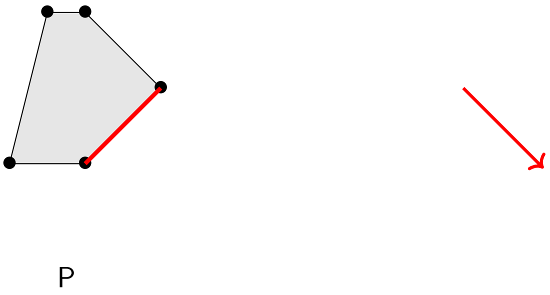
Face: $\mathrm{P}^{\boldsymbol{c}} := \left\{\boldsymbol{x} \in \mathbb{R}^n \ ; \ \langle \boldsymbol{x}, \boldsymbol{c}\rangle = \max_{\boldsymbol{y}\in\mathrm{P}} \langle \boldsymbol{y}, \boldsymbol{c}\rangle\right\}$



Faces

Definition

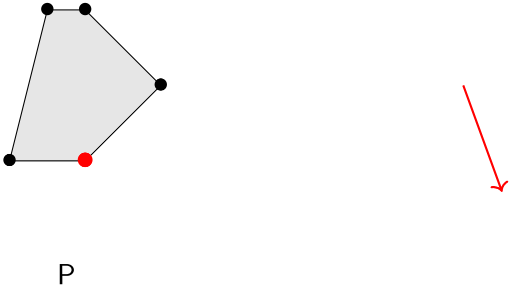
Face: $\mathrm{P}^{\boldsymbol{c}} := \left\{ \boldsymbol{x} \in \mathbb{R}^n \ ; \ \langle \boldsymbol{x}, \boldsymbol{c} \rangle = \max_{\boldsymbol{y} \in \mathrm{P}} \langle \boldsymbol{y}, \boldsymbol{c} \rangle \right\}$



Faces

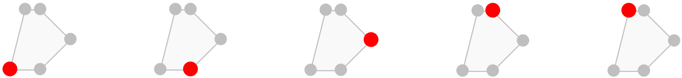
Definition

Face: $\mathrm{P}^{\boldsymbol{c}} := \left\{ \boldsymbol{x} \in \mathbb{R}^n \;;\; \langle \boldsymbol{x}, \boldsymbol{c} \rangle = \max_{\boldsymbol{y} \in \mathrm{P}} \langle \boldsymbol{y}, \boldsymbol{c} \rangle \right\}$



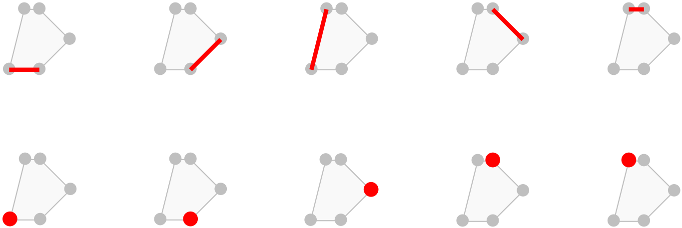
Face lattice

Face lattice: poset of inclusions of faces

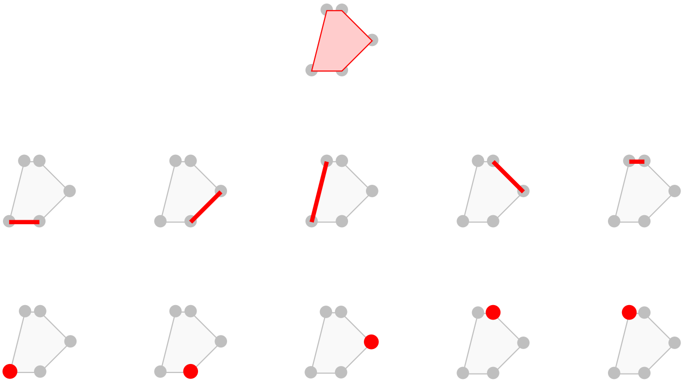


Face lattice

Face lattice: poset of inclusions of faces

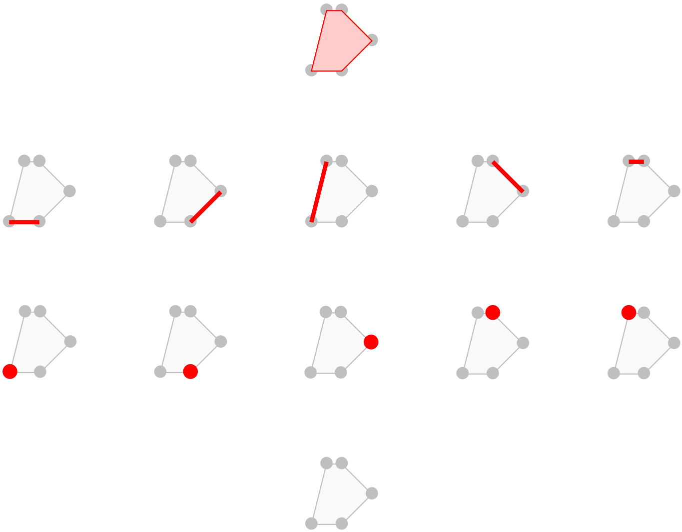


Face lattice: poset of inclusions of faces



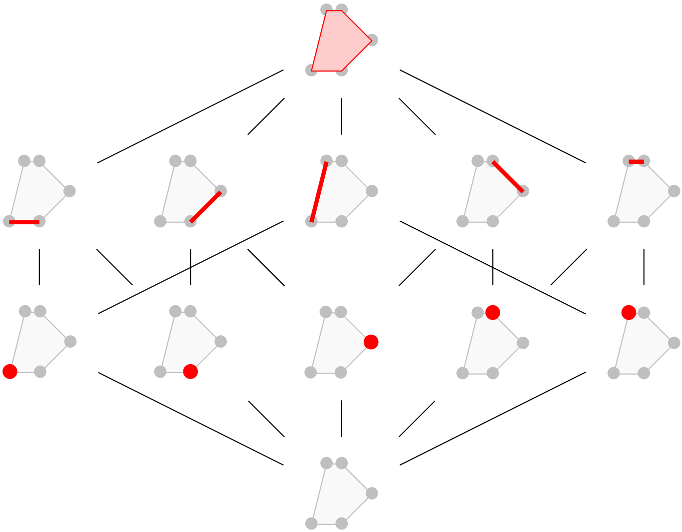
Face lattice

Face lattice: poset of inclusions of faces



Face lattice

Face lattice: poset of inclusions of faces



Deformations (weak Minkowski summands)

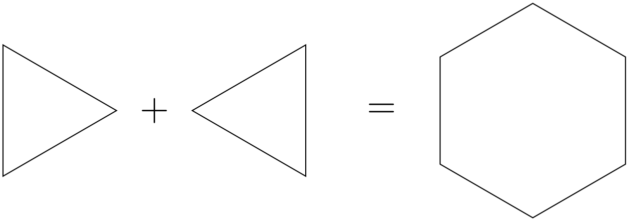
Minkowski sum

Definition

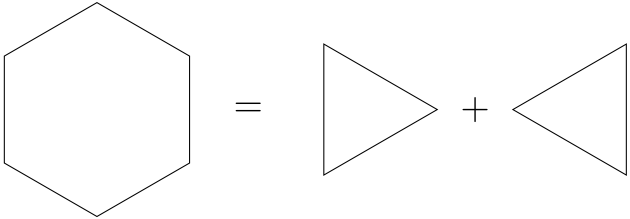
P, Q polytopes. *Minkowski sum*:

$$\mathrm{P} + \mathrm{Q} = \{\boldsymbol{p} + \boldsymbol{q} \;\; ; \;\; \boldsymbol{p} \in \mathrm{P}, \;\; \boldsymbol{q} \in \mathrm{Q}\}$$

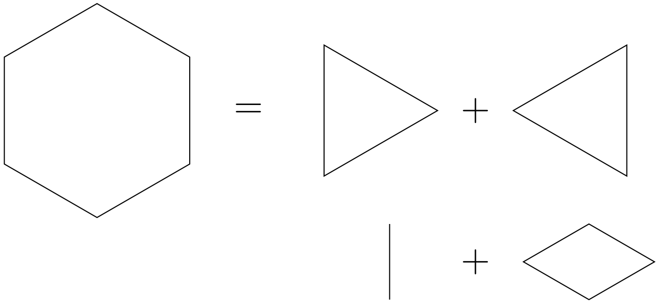
N.B. $\mathrm{Vert}(\mathrm{P} + \mathrm{Q}) \subseteq \mathrm{Vert}(\mathrm{P}) + \mathrm{Vert}(\mathrm{Q})$



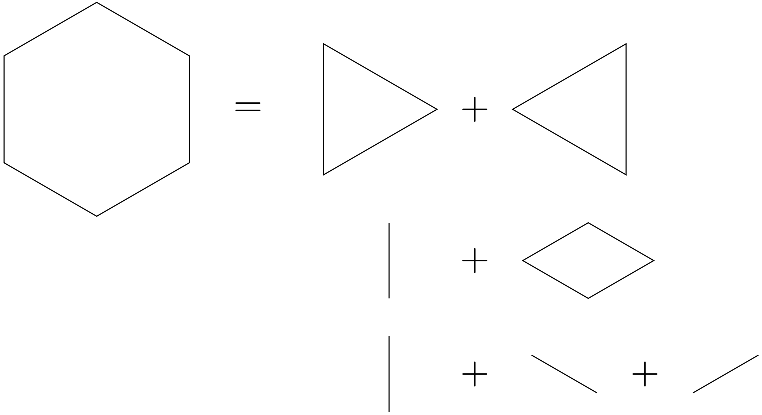
Minkowski sum



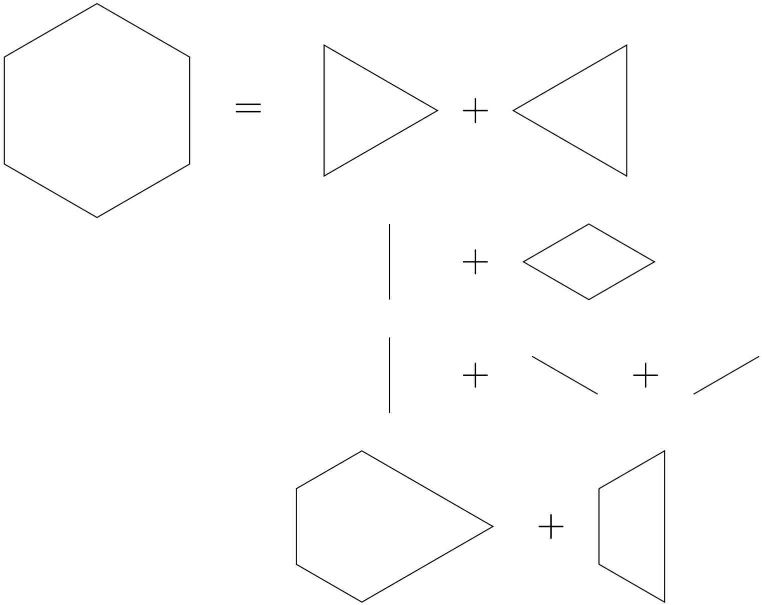
Minkowski sum



Minkowski sum



Minkowski sum



Minkowski summands

Definition

Q is a *Minkowski summand*, a.k.a. *deformation*, of P when there exists R and $\lambda > 0$ with:

$$\lambda \mathsf{P} = \mathsf{Q} + \mathsf{R}$$

Minkowski summands

Definition

Q is a *Minkowski summand*, a.k.a. *deformation*, of P when there exists R and $\lambda > 0$ with:

$$\lambda \mathrm{P} = \mathrm{Q} + \mathrm{R}$$

Deformation cone: $\mathbb{DC}(\mathrm{P}) = \{\mathrm{Q} \ ; \ \mathrm{Q} \text{ is a deformation of } \mathrm{P}\}$
Indecomposable: deformations of P are translated-dilations of P

Minkowski summands

Definition

Q is a *Minkowski summand*, a.k.a. *deformation*, of P when there exists R and $\lambda > 0$ with:

$$\lambda \mathsf{P} = \mathsf{Q} + \mathsf{R}$$

Deformation cone: $\mathbb{DC}(\mathsf{P}) = \{\mathsf{Q} \; ; \; \mathsf{Q} \text{ is a deformation of } \mathsf{P}\}$
Indecomposable: deformations of P are translated-dilations of P

What is the best way to write P as a Minkowski sum ?

- With the fewest number of (indecomposable) summands ?
- With the (indecomposable) summands of smallest dimension ?
- Respecting some symmetries ?
- . . .

Definition

Q is a *Minkowski summand*, a.k.a. *deformation*, of P when there exists R and $\lambda > 0$ with:

$$\lambda P = Q + R$$

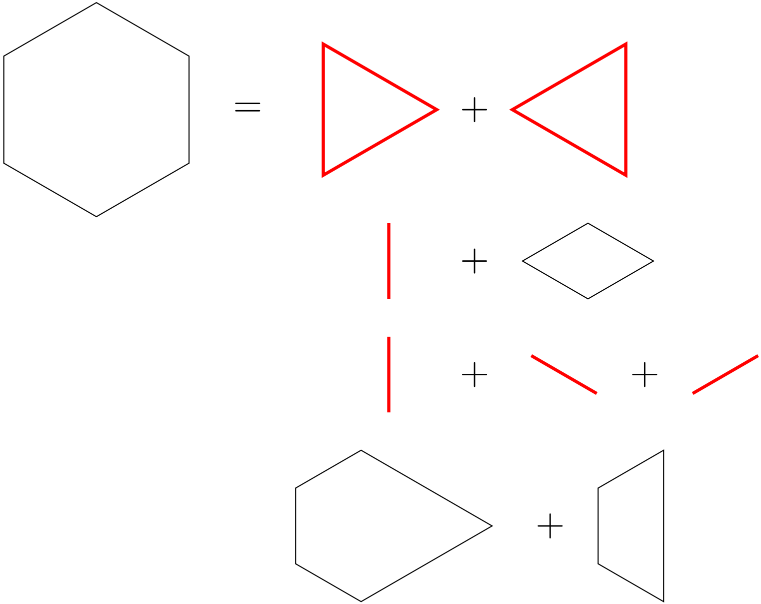
Deformation cone: $\mathbb{DC}(P) = \{Q \ ; \ Q \text{ is a deformation of } P\}$
Indecomposable: deformations of P are translated-dilations of P

What is the best way to write P as a Minkowski sum ?

- With the fewest number of (indecomposable) summands ?
- With the (indecomposable) summands of smallest dimension ?
- Respecting some symmetries ?
- ...

$\Longrightarrow$ What is the structure of $\mathbb{DC}(P)$?

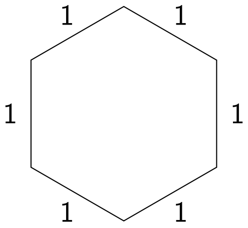
Minkowski summands

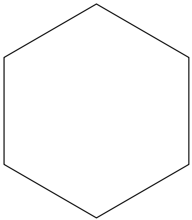


Deformation

Observation

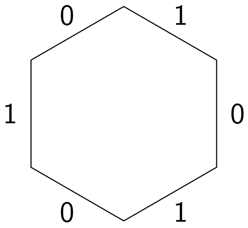
If $P = Q + R$, then the edges of P "are" edges of Q or of R.
$\Rightarrow$ I can write deformations of P using edge-length vectors.

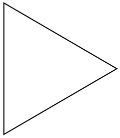




Observation

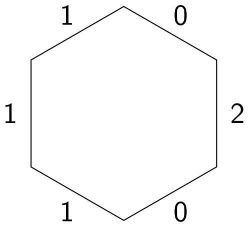
If $P = Q + R$, then the edges of P "are" edges of Q or of R.
$\Rightarrow$ I can write deformations of P using edge-length vectors.

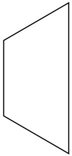




Observation

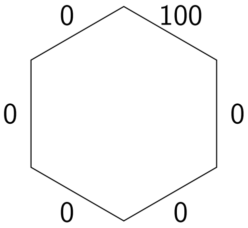
If $P = Q + R$, then the edges of P "are" edges of Q or of R.
$\Rightarrow$ I can write deformations of P using edge-length vectors.

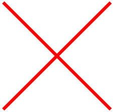




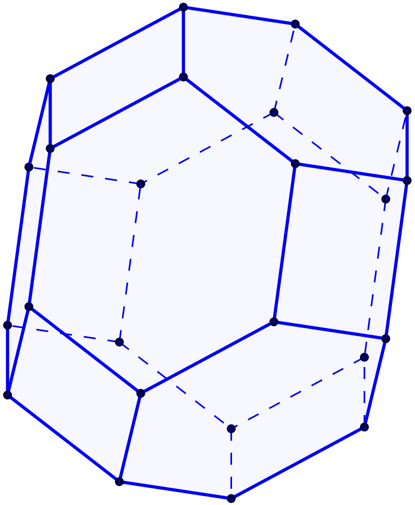
Observation

If $P = Q + R$, then the edges of P "are" edges of Q or of R.
$\Rightarrow$ I can write deformations of P using edge-length vectors.

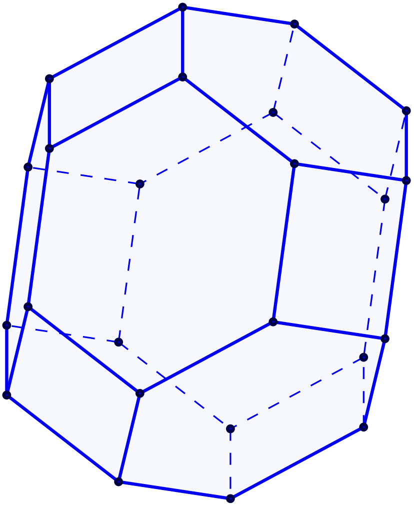




Deformations of 3-dim permutahedron



Permutahedron Π_4



Sequence of deformations of Π_4

Edge-length deformation cone

Theorem

Q *deformation of* P $\Leftrightarrow$ *same edge-directions, but different lengths*

Definition

Edge-length deformation cone: $\mathbb{DC}(\mathrm{P}) = \{\mathrm{Q}\ ;\ \mathrm{Q}\ \text{same edge-dir}\ \mathrm{P}\}$

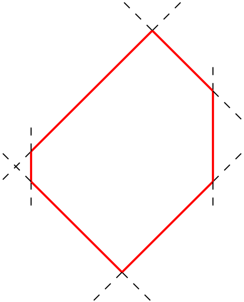
Edge-length deformation cone

Theorem

Q *deformation of* P $\Leftrightarrow$ *same edge-directions, but different lengths*

Definition

Edge-length deformation cone: $\mathbb{DC}(\mathrm{P}) = \{\mathrm{Q} \; ; \; \mathrm{Q} \text{ same edge-dir } \mathrm{P}\}$



Parametrization:

edge-length vector:

$\boldsymbol{\ell} = (\ell_{\boldsymbol{e}})_{\boldsymbol{e} \text{ edge}}$

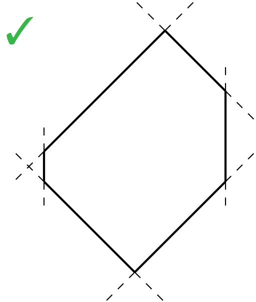
Edge-length deformation cone

Theorem

Q *deformation of* P $\Leftrightarrow$ *same edge-directions, but different lengths*

Definition

Edge-length deformation cone: $\mathbb{DC}(\mathrm{P}) = \{\mathrm{Q} \ ; \ \mathrm{Q} \text{ same edge-dir } \mathrm{P}\}$



Parametrization:

edge-length vector:

$\boldsymbol{\ell} = (\ell_{\boldsymbol{e}})_{\boldsymbol{e} \text{ edge}}$

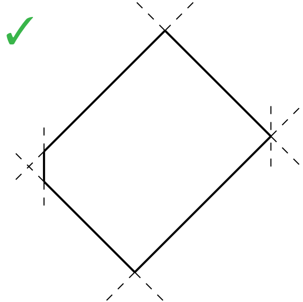
Edge-length deformation cone

Theorem

Q *deformation of* P $\Leftrightarrow$ *same edge-directions, but different lengths*

Definition

Edge-length deformation cone: $\mathbb{DC}(\mathrm{P}) = \{\mathrm{Q} \;;\; \mathrm{Q} \text{ same edge-dir } \mathrm{P}\}$



Parametrization:

edge-length vector:

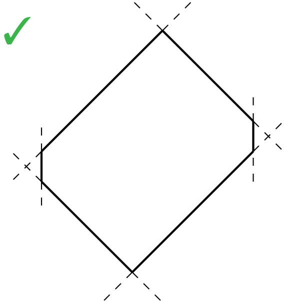
$\boldsymbol{\ell} = (\ell_{\boldsymbol{e}})_{\boldsymbol{e} \text{ edge}}$

Theorem

Q *deformation of* P $\Leftrightarrow$ *same edge-directions, but different lengths*

Definition

Edge-length deformation cone: $\mathbb{DC}(\mathrm{P}) = \{\mathrm{Q}\ ;\ \mathrm{Q} \text{ same edge-dir } \mathrm{P}\}$



Parametrization:

edge-length vector:
$\boldsymbol{\ell} = (\ell_{\boldsymbol{e}})_{\boldsymbol{e} \text{ edge}}$

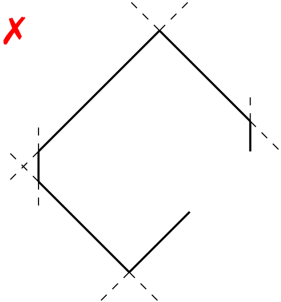
Edge-length deformation cone

Theorem

Q *deformation of* P $\Leftrightarrow$ *same edge-directions, but different lengths*

Definition

Edge-length deformation cone: $\mathbb{DC}(\mathrm{P}) = \{\mathrm{Q}\ ;\ \mathrm{Q}\text{ same edge-dir P}\}$



Parametrization:

edge-length vector:

$\boldsymbol{\ell} = (\ell_{\boldsymbol{e}})_{\boldsymbol{e}\text{ edge}}$

Cycle equations:

linear equations on $\boldsymbol{\ell}$

$\ell_{\boldsymbol{e}} \geq 0$ for all $\boldsymbol{e}$ edge

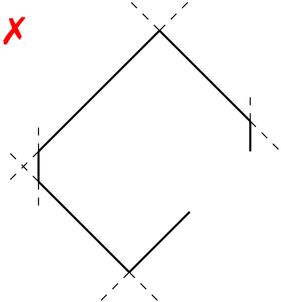
Edge-length deformation cone

Theorem

Q *deformation of* P $\Leftrightarrow$ *same edge-directions, but different lengths*

Definition

Edge-length deformation cone: $\mathbb{DC}(\mathrm{P}) = \{\mathrm{Q}\ ;\ \mathrm{Q} \text{ same edge-dir } \mathrm{P}\}$



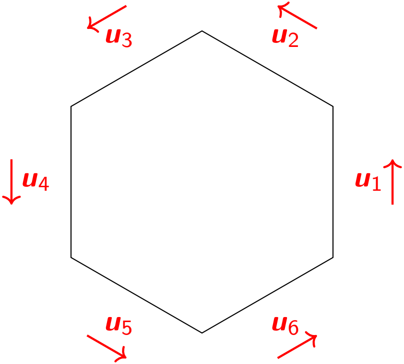
Parametrization:

edge-length vector:
$\boldsymbol{\ell} = (\ell_{\boldsymbol{e}})_{\boldsymbol{e} \text{ edge}}$

Cycle equations:
linear equations on $\boldsymbol{\ell}$
$\ell_{\boldsymbol{e}} \geq 0$ for all $\boldsymbol{e}$ edge

$\mathrm{P}_{\boldsymbol{\ell}}$ = start at a vertex, find the coordinates of the other vertices from the graph of P and $\boldsymbol{\ell}$

Cycle equations



For F a 2-dim face of P:

$$\sum_{\boldsymbol{e}} \boldsymbol{u}_{\boldsymbol{e}} = \mathbf{0} \quad , \quad \boldsymbol{u}_{\boldsymbol{e}} \text{ unit vector}$$

hence

$$\sum_{\boldsymbol{e}} \ell_{\boldsymbol{e}} \boldsymbol{u}_{\boldsymbol{e}} = \mathbf{0}$$

Summary on $\mathbb{DC}(\mathrm{P})$

$\mathbb{DC}(\mathrm{P})$

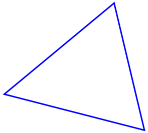
Q Minkowski summands	$\boldsymbol{\ell}$ edge-lengths	$\boldsymbol{h}$ heights on rays
$\mathrm{Q}_1 + \mathrm{Q}_2$ Dilation $\lambda \mathrm{Q}$ Translations	$\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2$ $\lambda \boldsymbol{\ell}$ Has been fixed	$\boldsymbol{h}_1 + \boldsymbol{h}_2$ $\lambda \boldsymbol{h}$ Lineal
complicated	edge directions Cycle equations V-description	normal fan $\mathcal{N}_\mathrm{P}$ Wall-crossing ineq. H-description
Polytope algebra	Weight algebra	Polynomial algebra

$\mathbb{DC}(\mathrm{P})$ is a ray = P Minkowski indecomposable

$\mathbb{DC}(\mathrm{P})$ is simplicial cone = P has **unique** Minkowski decomposition

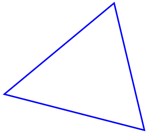
Graph of edge dependencies

Cycle equation of a triangle, quadrilateral



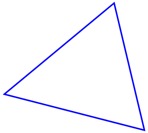
Triangle:
3 variables (= lengths of the edges)

Cycle equation of a triangle, quadrilateral



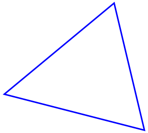
Triangle:
3 variables (= lengths of the edges)
2 equations (= 1 equation in dimension 2)

Cycle equation of a triangle, quadrilateral



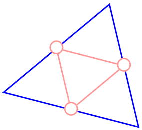
Triangle:
3 variables (= lengths of the edges)
2 equations (= 1 equation in dimension 2)
$\Rightarrow$ dim(space of solutions) = 1

Cycle equation of a triangle, quadrilateral



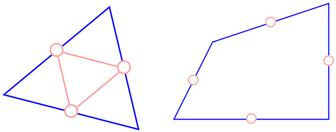
Triangle:
3 variables (= lengths of the edges)
2 equations (= 1 equation in dimension 2)
$\Rightarrow$ dim(space of solutions) $= 1$
$\Rightarrow$ Indecomposable

Cycle equation of a triangle, quadrilateral



Triangle:
3 variables (= lengths of the edges)
2 equations (= 1 equation in dimension 2)
$\Rightarrow$ dim(space of solutions) = 1
$\Rightarrow$ Indecomposable
$\Rightarrow$ If I know the length of 1 edge, I know the length of the others.

Cycle equation of a triangle, quadrilateral



Triangle:

3 variables (= lengths of the edges)

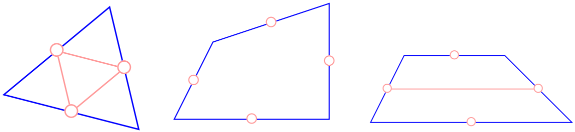
2 equations (= 1 equation in dimension 2)

$\Rightarrow$ dim(space of solutions) = 1

$\Rightarrow$ Indecomposable

$\Rightarrow$ If I know the length of 1 edge, I know the length of the others.

Cycle equation of a triangle, quadrilateral



Triangle:

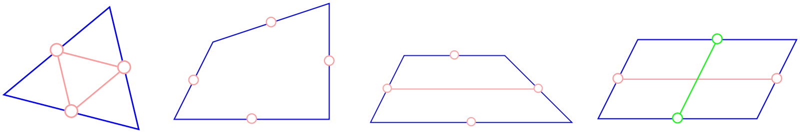
3 variables (= lengths of the edges)

2 equations (= 1 equation in dimension 2)

$\Rightarrow$ dim(space of solutions) = 1

$\Rightarrow$ Indecomposable

$\Rightarrow$ If I know the length of 1 edge, I know the length of the others.



Triangle:

3 variables (= lengths of the edges)

2 equations (= 1 equation in dimension 2)

$\Rightarrow$ dim(space of solutions) = 1

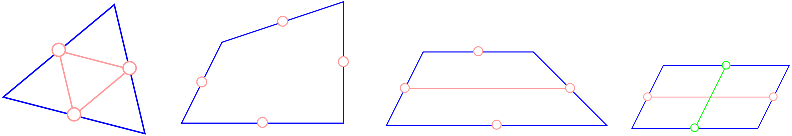
$\Rightarrow$ Indecomposable

$\Rightarrow$ If I know the length of 1 edge, I know the length of the others.

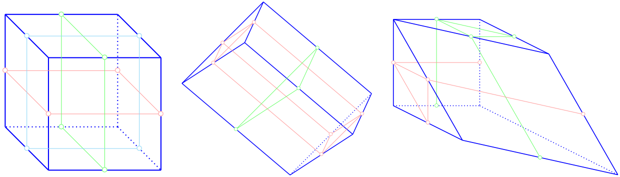
Graph of edge dependencies ED(P):

nodes: edges of P

arcs: link two *dependent* edges, i.e. I can deduce the length of one from the length of the other, using the cycle equations



(a) Triangle (b) Quadrilateral (c) Trapezoid (d) Parallelogram



(e) Cube (f) Prism (g) Hemicube (h) Pyramid

Problem 1:
How to find edges of $ED(\mathrm{P})$.

The two problems

Problem 1:
How to find edges of $ED(\mathrm{P})$.

Problem 2:
If I have enough edges in $ED(\mathrm{P})$, what can I conclude on $\mathbb{DC}(\mathrm{P})$ or on the indecomposability of P?

The two problems

Problem 1:
How to find edges of $ED(\mathrm{P})$.

Problem 2:
If I have enough edges in $ED(\mathrm{P})$, what can I conclude on $\mathbb{DC}(\mathrm{P})$ or on the indecomposability of P?

A Gale's criterion, '54

All 2-faces of P are triangles $\Rightarrow$ P indecomposable.

The two problems

Problem 1:
How to find edges of $ED(\mathrm{P})$.

Problem 2:
If I have enough edges in $ED(\mathrm{P})$, what can I conclude on $\mathbb{DC}(\mathrm{P})$ or on the indecomposability of P?

A Gale's criterion, '54

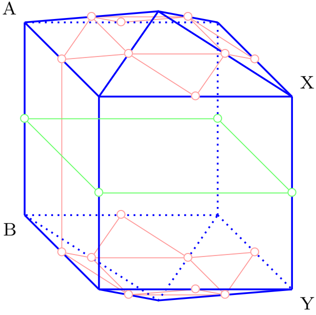
All 2-faces of P are triangles $\Rightarrow$ P indecomposable.

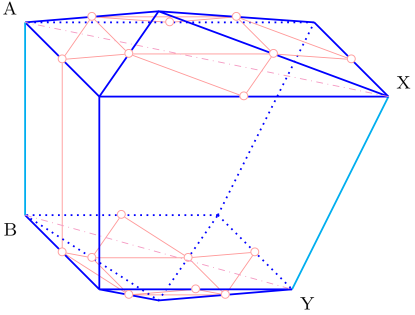
"triangles" $\rightarrow$ edges in $ED(\mathrm{P})$
"all 2-faces" $\rightarrow$ enough edges in $ED(\mathrm{P})$

Problem 1: creating edges

In $ED(\mathrm{P})$, two nodes e, f ($\in$ edges of P) are linked if, e.g.:

- in a common triangle (not necessarily a 2 -face),

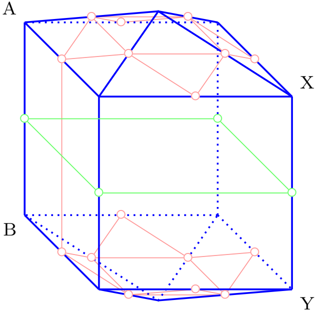


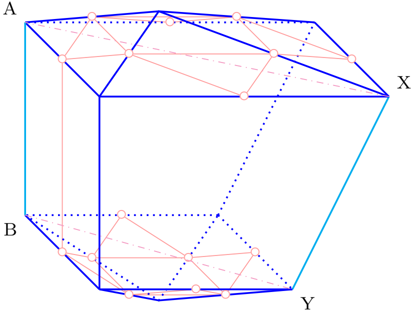


In $ED(\mathrm{P})$, two nodes e, f ($\in$ edges of P) are linked if, e.g.:

- in a common triangle (not necessarily a 2 -face),

- are opposite in a trapezoid,



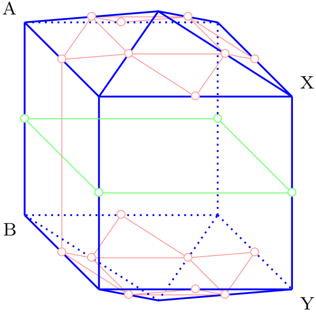


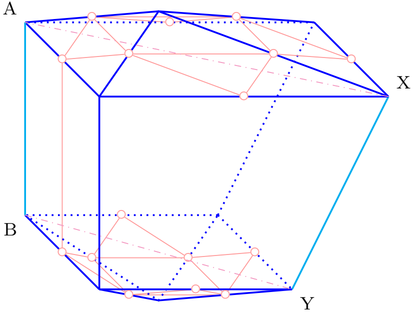
Problem 1: creating edges

In $ED(\mathrm{P})$, two nodes e, f ($\in$ edges of P) are linked if, e.g.:

- in a common triangle (not necessarily a 2 -face),
- belong to a cycle of edges which spans a full-dim space,

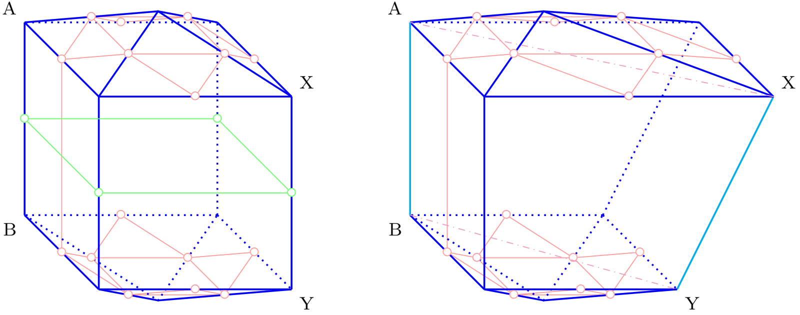
- are opposite in a trapezoid,





In $ED(\mathrm{P})$, two nodes e, f ($\in$ edges of P) are linked if, e.g.:

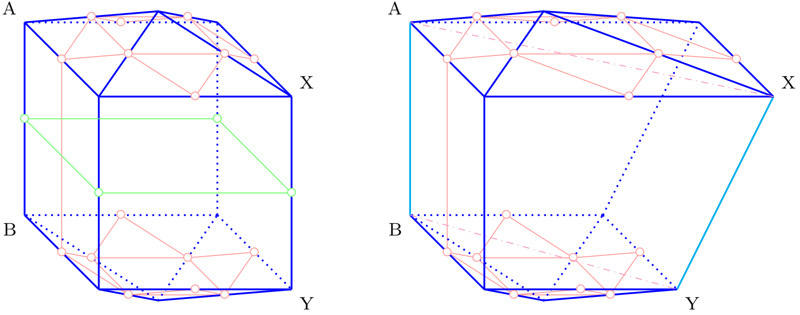
- in a common triangle (not necessarily a 2 -face),
- belong to a cycle of edges which spans a full-dim space,
- belong to an indecomposable sub-graph of P,
- are opposite in a trapezoid,



Problem 1: creating edges

In $ED(\mathsf{P})$, two nodes e, f ($\in$ edges of P) are linked if, e.g.:

- in a common triangle (not necessarily a 2 -face),
- belong to a cycle of edges which spans a full-dim space,
- belong to an indecomposable sub-graph of P,
- are opposite in a trapezoid,
- are in a sub-graph of P which projects down to indecomposable,

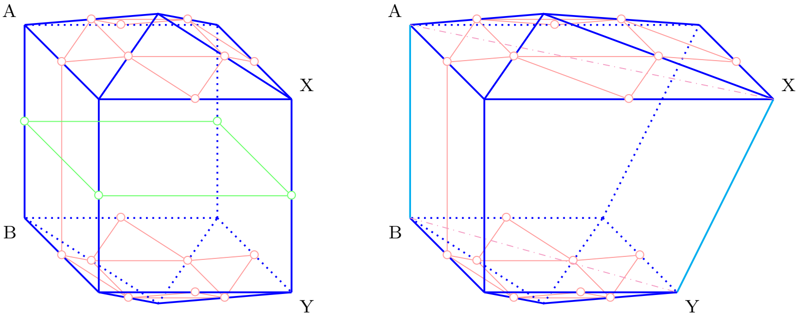


Problem 1: creating edges

In $ED(\mathsf{P})$, two nodes e, f ($\in$ edges of P) are linked if, e.g.:

- in a common triangle (not necessarily a 2 -face),
- belong to a cycle of edges which spans a full-dim space,
- belong to an indecomposable sub-graph of P,
- are opposite in a trapezoid,
- are in a sub-graph of P which projects down to indecomposable,
- ... (there is a nicer way to write these properties)

+ implicit edges



Problem 2: consequences of $ED(\mathrm{P})$

Theorem (Padrol–P. '25)

If $ED(\mathrm{P})$ is connected, then P is indecomposable

Theorem (Padrol–P. '25)

If $ED(\mathrm{P})$ is connected, then P is indecomposable

$S \subseteq V(\mathrm{P})$ is *dependent* if for all $u, v \in S$, there is a path (in $G(\mathrm{P})$) of dependent edges between u and v.

Theorem (Padrol–P. '25)

If there exists a set S of dependent vertices such that every facet contains a vertex of S, then P is indecomposable.

Problem 2: consequences of $ED(\mathrm{P})$

Theorem (Padrol–P. '25)

If $ED(\mathrm{P})$ is connected, then P is indecomposable

$S \subseteq V(\mathrm{P})$ is *dependent* if for all $u, v \in S$, there is a path (in $G(\mathrm{P})$) of dependent edges between u and v.

Theorem (Padrol–P. '25)

If there exists a set S of dependent vertices such that every facet contains a vertex of S, then P is indecomposable.

Corollaries: Gale '54, Shepard '63, Kallay '82, McMullen '87, Yost–Przesławski '08 & '16 criteria...

Corollary (Padrol–P. '25)

If there is $S \subseteq V(\mathrm{P})$*, and* $X_1, \ldots, X_r$ *dependent sets of edges with*

- *any two vertices of* S *are connected via* $\bigcup_i X_i$*, and*
- *every facet of* P *contains a vertex of* S*,*

then: $\dim \mathbb{DC}(\mathrm{P}) \leq r$.

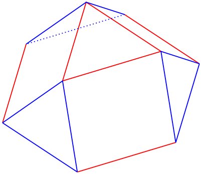
Beyond

Corollary (Padrol–P. '25)

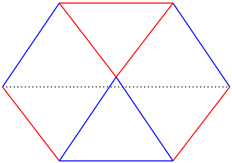
If there is $S \subseteq V(\mathrm{P})$, and $X_1, \dots, X_r$ dependent sets of edges with

- *any two vertices of S are connected via $\bigcup_i X_i$, and*
- *every facet of P contains a vertex of S,*

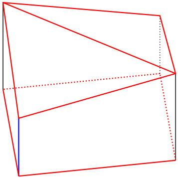
then: $\dim \mathbb{DC}(\mathrm{P}) \leq r$.



(a) Triang. cupola

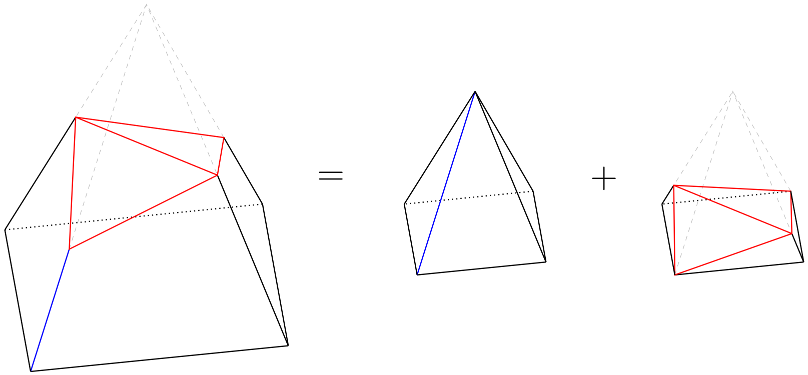


(b) Sum of 2 triangles



(c) Chiseled cube

$\dim \mathbb{DC}(\mathrm{P}) = 2$, only one Minkowski decomposition (in 2 terms)



$\dim \mathbb{DC}(P) = 2$, this is the only Minkowski decomposition

New indecomposable generalized permutahedra

Generalized permutahedra: edge directions are $\boldsymbol{e}_i - \boldsymbol{e}_j$ for $i \neq j$
Edmonds problem '70: find all indecomposable gene. permut.

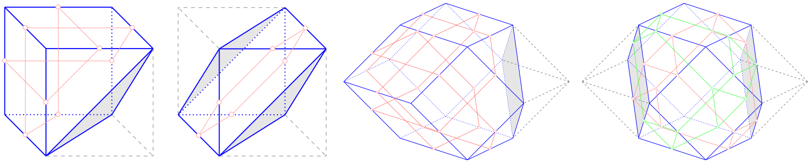
New indecomposable generalized permutahedra

Generalized permutahedra: edge directions are $\boldsymbol{e}_i - \boldsymbol{e}_j$ for $i \neq j$
Edmonds problem '70: find all indecomposable gene. permut.

The *graphical zonotope* of $G = (V, E)$ is: $\mathrm{Z}_G = \sum_{i\,j \in E} [\boldsymbol{e}_i, \boldsymbol{e}_j]$

Theorem (Padrol–P. '25)

For all complete bipartite graphs $K_{n,m}$ (except $n = m = 2$), it is possible to truncate 1 or 2 vertices of $\mathrm{Z}_{K_{n,m}}$ to obtain an indecomposable generalized permutahedron.



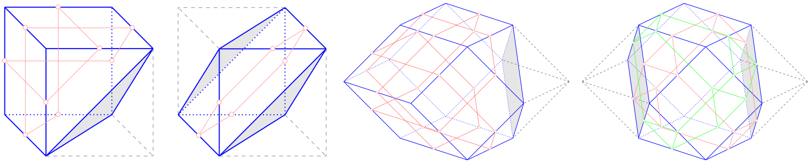
New indecomposable generalized permutahedra

Generalized permutahedra: edge directions are $\boldsymbol{e}_i - \boldsymbol{e}_j$ for $i \neq j$
Edmonds problem '70: find all indecomposable gene. permut.

The *graphical zonotope* of $G = (V, E)$ is: $\mathrm{Z}_G = \sum_{i\,j \in E} [\boldsymbol{e}_i, \boldsymbol{e}_j]$

Theorem (Padrol–P. '25)

For all complete bipartite graphs $K_{n,m}$ (except $n = m = 2$), it is possible to truncate 1 or 2 vertices of $\mathrm{Z}_{K_{n,m}}$ to obtain an indecomposable generalized permutahedron.



This gives $2\left\lfloor \frac{n-1}{2} \right\rfloor$ new indecomposable gene. permutahedra

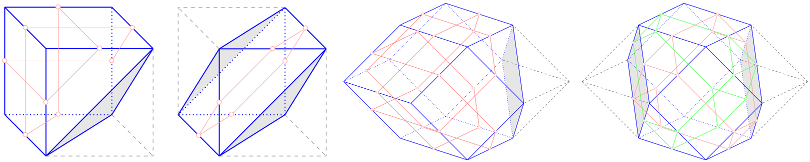
New indecomposable generalized permutahedra

Generalized permutahedra: edge directions are $\boldsymbol{e}_i - \boldsymbol{e}_j$ for $i \neq j$
Edmonds problem '70: find all indecomposable gene. permut.

The *graphical zonotope* of $G = (V, E)$ is: $Z_G = \sum_{ij \in E} [\boldsymbol{e}_i, \boldsymbol{e}_j]$

Theorem (Padrol–P. '25)

For all complete bipartite graphs $K_{n,m}$ (except $n = m = 2$), it is possible to truncate 1 or 2 vertices of $Z_{K_{n,m}}$ to obtain an indecomposable generalized permutahedron.



This gives $2\left\lfloor \frac{n-1}{2} \right\rfloor$ new indecomposable gene. permutahedra
Loho–Padrol–P.'25: we create $2^{2^{n-2}}$ indecomposable gene. permut.

Besides

Other applications:

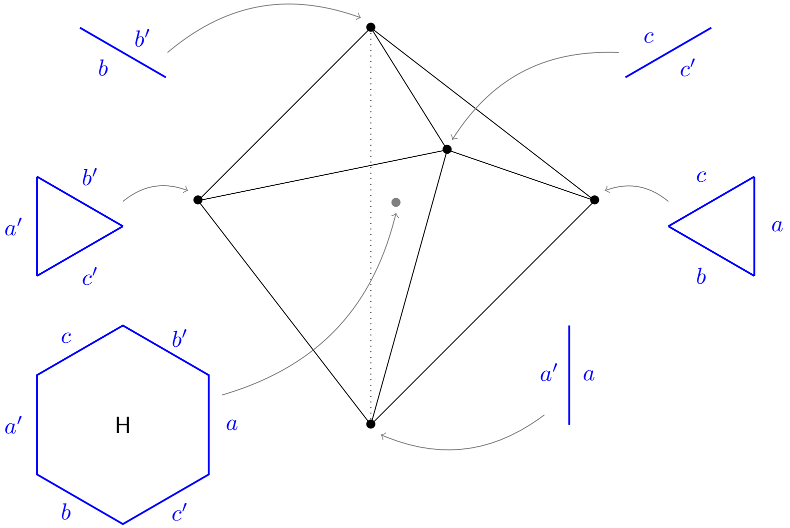
- parallelogramic Minkowski sums, product of polytopes
- "autonomous and dependent" parts of $ED(\mathrm{P})$ give rays of $\mathbb{DC}(\mathrm{P})$
- you can stack/delete vertices on polytopes, study indecomposability
- you can deal with zonotopes whose 2-faces are parallelograms
- new proof: matroid pol. indecomposable $\Leftrightarrow$ matroid connected
- you use combinatorics on regular polytopes instead of geometry
- ...

Besides

Other applications:

- parallelogramic Minkowski sums, product of polytopes
- "autonomous and dependent" parts of $ED(\mathrm{P})$ give rays of $\mathbb{DC}(\mathrm{P})$
- you can stack/delete vertices on polytopes, study indecomposability
- you can deal with zonotopes whose 2-faces are parallelograms
- new proof: matroid pol. indecomposable $\Leftrightarrow$ matroid connected
- you use combinatorics on regular polytopes instead of geometry
- ...

Be careful, $ED(\mathrm{P})$ is not always sufficient for recovering $\mathbb{DC}(\mathrm{P})$.



Merci ! Thank you!

Bonus slides...

What else do I do?

Submodular cone: "Many rays of the submodular cone"

Linear programming: "Vertices of the monotone path polytopes of hypersimplicies" & "Pivot polytopes of products of simplices and shuffles of associahedra"

Random polytopes: "Unimodality of the number of paths per length on polytopes: Examples, counter-examples, and central limit theorem"

Ehrhart theory, triangulations: "Ehrhart non-positivity and unimodular triangulations for classes of s-lecture hall simplices"

3D polytopes, framing lattices, polytope/weight algebra, hypergraphs, Grey codes,...

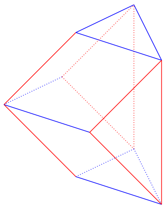
Anything that might contain a polytope somewhere is interesting !!

Parallelogramic Minkowski sums

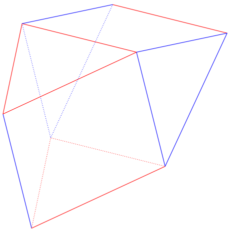
P, Q are in *parallelogramic position* if no edge of P is in a plane spanned by the 2-faces of Q

Theorem (Padrol–P. '25)

P, Q *in parallelogramic position:* $\mathbb{DC}(P + Q) \simeq \mathbb{DC}(P) \times \mathbb{DC}(Q)$.



(a) Diminished trapezohedron



(b) Gyrobifastigium

Corollary (Padrol–P. '25)

$\mathbb{DC}(P \times Q) \simeq \mathbb{DC}(P) \times \mathbb{DC}(Q)$